Определим множества

$$0_\gamma = \{\, y \in B \mid (y,0) \in \upsilon(\gamma) \,\}, \quad 0^*_\gamma = 0^\perp_{1-\gamma} = \{\, g \in B^* \mid \forall\, y \in 0_{1-\gamma}\ gy = 0 \,\}.$$

Зададим в сопряженном пространстве B^* систему $\mathfrak{V}^*$ отношений эквивалентности, считая функционалы f, g $\upsilon^*(\gamma)$-эквивалентными, если $f - g \in 0^*_\gamma$. Система $\mathfrak{V}^*$ отношений эквивалентности элементов пространства B^* обладает всеми перечисленными выше свойствами.

Т е о р е м а. *Если линейный оператор $F : B \to B$ вольтерров на системе $\mathfrak{V}$, то сопряженный оператор $F^* : B^* \to B^*$ будет вольтерровым на системе $\mathfrak{V}^*$.*

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POSITIVE INVARIANCE AND PERIODIC SOLUTIONS FOR DIFFERENTIAL INCLUSION WITH NON-CONVEX RIGHT-HAND SIDE

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The talk is concerned with the non-autonomous ordinary differential inclusion in finite dimensional space with periodic, compact, but non necessarily convex valued right-hand side. It is shown that if for such an inclusion there exists a strongly positively invariant set $\mathfrak{M}$, then there exists a periodic solution of the inclusion which stays in $\mathfrak{M}$.

Let $\mathbb{R}^n$ be a Euclidian space with the scalar product $\langle x, y\rangle$, $x, y \in \mathbb{R}^n$, usual norm $|x| = \sqrt{\langle x, y\rangle}$, and metric $\rho(x,y) = |x-y|$, and let comp$(\mathbb{R}^n)$ stand for a set of all compact subsets of $\mathbb{R}^n$. By $AC([t_0,t_1],\mathbb{R}^n)$ we denote the space of all absolutely continuous functions $x : [t_0,t_1] \to \mathbb{R}^n$ with the norm $\|x\|_{AC} = |x(t_0)| + \int_{t_0}^{t_1} |\dot{x}(t)|dt$.

Consider an ordinary differential inclusion

$$\dot{x} \in F(t,x), \tag{1}$$

where $F : \mathbb{R} \times \mathbb{R}^n \to \text{comp}(\mathbb{R}^n)$ satisfies Caratheodory conditions. As a *solution of* (1) on an interval $I \subset \mathbb{R}$ we suppose a function $x \in AC(I,\mathbb{R}^n)$ satisfying inclusion (1) for a.e. $t \in I$, so we deal with the Caratheodory type solutions.

Let a map $M : \mathbb{R} \to \text{comp}(\mathbb{R}^n)$ be continuous and denote a set

$$\mathfrak{M} \doteq \{(t,x) \in \mathbb{R} \times \mathbb{R}^n : x \in M(t)\}, \tag{2}$$

which represents the graph of M. The set $\mathfrak{M}$ is called *strongly positively invariant under inclusion* (1) if for every point $z_0 = (t_0, x_0) \in \mathfrak{M}$ any solution $t \to x(t, z_0)$ of the Cauchy problem for (1) with initial condition $x(t_0) = x_0$ satisfies $(t, x(t, z_0)) \in \mathfrak{M}$ for every $t \geqslant t_0$.

The sufficient conditions for the set $\mathfrak{M}$ to be strongly positively invariant under inclusion can be expressed in terms of so-called Lyapunov functions. Let $r > 0$ and let $\mathfrak{M}^r \doteq \{(t,x) \in \mathbb{R}\times\mathbb{R}^n : x \in M^r(t)\}$ denote a closed r-neighborhood of the set $\mathfrak{M}$. We say that continuous function $V : \mathfrak{M}^r \to \mathbb{R}$ is a *Lyapunov function with respect to the set* $\mathfrak{M}$ if $V(t,x) = 0$ for $(t,x) \in \partial\mathfrak{M}$ and $V(t,x) > 0$ for $(t,x) \in \mathfrak{M}^r\backslash\mathfrak{M}$. If the Lyapunov function V in addition is locally lipschitz, we can consider the generalized Clarke derivative (see, e.g. [1]) of V at the point (t,x) in the direction $(1,h) \in \mathbb{R}\times\mathbb{R}^n$ which is defined as follows:

$$V^o(t,x;h) \doteq \limsup_{\substack{(\vartheta,y)\to(t,x)\\ \delta\to 0+}} \frac{V(\vartheta+\delta, y+\delta h) - V(\vartheta,y)}{\delta}.$$

The relation $V_F^o(t,x) \doteq \max\limits_{h\in F(t,x)} V^o(t,x;h)$ we will call the *derivative of function V with respect to inclusion* (1).

Theorem 1. *Let us have a Lyapunov function $(t,x) \to V(t,x)$, $(t,x) \in \mathfrak{M}^r$, which is locally lipschitz. If for some $\varepsilon \in (0,r]$ the inequality $V_F^o(t,x) \leqslant 0$ holds for any $(t,x) \in \mathfrak{M}^\varepsilon\backslash\mathfrak{M}$, then the set $\mathfrak{M}$ is strongly positively invariant.*

We suppose now an ordinary differential inclusion

$$\dot{x} \in F(t,x), \quad F(t+T,x) = F(t,x) \tag{3}$$

under the following assumptions:

(P1) $F : \mathbb{R}\times\mathbb{R}^n \to \mathrm{comp}(\mathbb{R}^n)$ satisfies Caratheodory conditions;

(P2) there exists continuous, T-periodic map $M : \mathbb{R} \to \mathrm{comp}(\mathbb{R}^n)$ such that $M(0)$ is convex and the corresponding set $\mathfrak{M}$ (see (2)) is strongly positively invariant under inclusion (3);

(P3) there exists an integrable function $k : \mathbb{R} \to \mathbb{R}_+$ such that for a.e. $t \in \mathbb{R}$ and each $x, y \in M(t)$

$$\mathrm{dist}(F(t,x), F(t,y)) \leqslant k(t)|x-y|.$$

Theorem 2. *Let the maps F and M satisfy the conditions $(P1)-(P3)$. Then there exists a periodic solution $t \to x(t)$ for the problem (3) such that $x(t) \in M(t)$ for all t.*

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К ПРОБЛЕМЕ ОБУЧЕНИЯ ДОКАЗАТЕЛЬСТВУ В КУРСЕ ГЕОМЕТРИИ ОСНОВНОЙ ШКОЛЫ

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В методической литературе для учителей математики, учебниках методики преподавания математики для педвузов выделяют следующие способы рассуждений при доказательстве (или поиске