2. *Беккенбах Э., Беллман Р.* Неравенства. М.: Мир, 1985. 280 с.

Abstract: On Virtinger's inequality as an example it is considerated using techniques, developed by Perm FDE seminar, to proof of integral inequalities at this article.

Key words: integran inequality; operator; W-substitution; quadratic summable function; conjugate operator; eigenvalue; spectral radius.

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ON THE SOLVABILITY OF RESONANCE BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH MONOTONE OPERATORS [1]

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Key words: periodic boudary value problem; resonance boudary value problem; functional differential equations; Favard constants, Green function.

Abstract: For a wide class of resonance boundary value problems for scalar functional differential equations with positive operators necessary and sufficient conditions of the unique solvability are obtained.

Periodic boundary value problems for different functional differential equations have attracted great attention during recent years (see [1–3] and lists of references). On the basic of the results of [2] some conditions of solvability for periodic problems were obtained in terms of maxima and minima of some polynomials. The optimality of solvability conditions and a recurrence relation for these maxima and minima were proved for all orders n only for some additional suppositions.

Here necessary and sufficient conditions of uniquely solvability for some classes of resonance boundary value problems (including periodic ones) are obtained.

Consider the boundary value problem for a linear scalar equation:

$$x^{(n)}(t) = (T^{+}x)(t) - (T^{-}x)(t) + f(t), \quad t \in [0,1], \tag{1}$$

$$\ell_i x = c_i, \ i = 1, \dots, n-1, \quad \ell_n x \equiv x^{(n-1)}(0) - x^{(n-1)}(1) = c_n, \tag{2}$$

where $n \geqslant 2$; $c_i \in R$, $i = 1, \dots, n$; $f \in L[0,1]$; the linear operators $T^{+/-} : C[0,1] \to L[0,1]$ are positive; the functionals

$$\ell_i x \equiv \sum_{j=0}^{n-1} \left(A_{ij} x^{(j)}(0) + B_{ij} x^{(j)}(1) \right), \ i = 1, \dots, n-1, \tag{3}$$

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are such that $\{x(t) = c,\ c \in R\}$ is the set of all solutions of the problem $x^{(n)} = 0$, $\ell_i x = 0$, $i = 1, \dots, n$. We will say that the prob;em is resonance since for $T^{+/-} = 0$, $f = 0$, $c_i = 0$, $i = 1, \dots, n$, the problem has nontrivial solutions.

Denote by $G(t,s)$ the Green function of the problem $x^{(n)} = f$, $\ell_i x = 0$, $i = 1, \dots, n-1$, $x(0) = 0$. Let the constants M_n, $n \geqslant 2$, be defined by the equalities

$$M_n = \max_{t_1, t_2 \in [0,1]} \left(\max_{s_1 \in [0,1]} \bigl(G(t_1, s_1) - G(t_2, s_1)\bigr) - \min_{s_2 \in [0,1]} \bigl(G(t_1, s_2) - G(t_2, s_2)\bigr) \right).$$

T h e o r e m 1. *Let nonnegative numbers $\mathcal{T}^+ \neq \mathcal{T}^-$ be given. Boundary value problem (1)–(2) is uniquely solvable for all positive operators $T^{+/-} : C[0,1] \to L[0,1]$ such that*

$$\int_0^1 (T^{+/-}1)(s)\, ds = \mathcal{T}^{+/-},$$

if and only if

$$\frac{Y}{1-Y} \leqslant X \leqslant 2(1 + \sqrt{1-Y}), \quad Y \leqslant \frac{3}{4},$$

where $X = M_n \max(\mathcal{T}^+, \mathcal{T}^-)$, $Y = M_n \min(\mathcal{T}^+, \mathcal{T}^-)$.

R e m a r k 1. The results of Theorem 1 are valid for much more general boundary conditions than (3) for some additional suppositions.

For the periodic problem for the n-th order equation we have: $M_n = |G(\frac{1}{2}, \frac{1}{2})|$ if n is even, $M_n = 2|G(\frac{1}{2}, \frac{1}{4})|$ if n is odd, where $G(t,s)$ is the Green function of the problem

$$x^{(n)} = f,\ x(0) = 0,\ x(1) = 0,\ x^{(i)}(0) = x^{(i)}(1),\ i = 1, \dots, n-2.$$

Moreover, the constants M_n coincide with the constants computed in [2] and possess the following properties:

$$M_n = \frac{F_{n-1}}{(2\pi)^{n-1}} = \begin{cases} (-1)^m \dfrac{E_{n-1}}{4^{n-1}\,(n-1)!}, & n = 2m+1, \\ (-1)^{m+1} 4(1 - \dfrac{1}{2^n}) B_n, & n = 2m, \end{cases}$$

where $F_n = \dfrac{4}{\pi} \displaystyle\sum_{k=0}^{\infty} \left(\frac{(-1)^k}{2k+1} \right)^{n+1}$ are the Favard constants [4, D. 27, P. 385], B_n are the Bernoulli numbers, E_n are the Euler numbers;
for even numbers n

$$M_n = \frac{4 C_{n,\infty}}{(2\pi)^{n-1}},$$

where $C_{n,\infty}$ are the «Stechkin constants» [4, D. 30, P. 385];
the sequence $\{M_n\}$ satisfies the following recurrent equalities

$$8\, n\, M_{n+1} = \sum_{k=1}^{n} M_k M_{n+1-k}, \quad M_1 = 1,$$

therefore, the sequence $\{M_n\}$ has a simple generating function

$$\sum_{n=0}^{\infty} M_{n+1} t^n = \frac{1}{\cos(t/4)} + \mathrm{tg}(t/4), \quad |t| < 2\pi;$$

$$M_{n+2} = \frac{A_n}{4^{n+1}(n+1)!},$$

where A_n are the numbers of up-down permutations of the numbers $\{0, 1, \ldots, n\}$ (see, for example, [5]).

REFERENCES

1. *Lomtatidze A.G., Půža B., Hakl R.* On periodic boundary value problems for first order functional differential equations // Differential equations. 2003. T. 39. № 3. P. 320–327. [In Russian]

2. *Hakl R., Mukhigulashvili S.* On one estimate for periodic functions// Georgian Math. J. 2005. V. 12. № 1. P. 97–114.

3. *Mukhigulashvili S.* On a periodic boundary value problem for third order linear functional differential equations // Nonlinear Anal. Theory, Methods, Appl., 2007. V. 66. № 2(A). P. 527–535.

4. *Hardy G.H., Littlewood J.E., Polya G.* Inequalities. Moscow: Inostrannaya Literatura, 1946. [Russian translation]

5. *Arnol'd V.I.* The calculus of snakes and the combinatorics of Bernoulli, Euler and Springer numbers of Coxeter groups // Russian Mathematical Surveys. 1992. V. 47. № 1(283). P. 3–45.

Аннотация: Для широкого класса резонансных краевых задач для скалярных функционально-дифференциальных уравнений с положительными операторами получены необходимые и достаточные условия однозначной разрешимости.

Ключевые слова: периодическая краевая задача; резонансная краевая задача; функционально-дифференциальные уравнения; константы Фавара; функция Грина.

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О РЕАЛИЗАЦИИ РАССТОЯНИЯ НА МНОЖЕСТВЕ РЕШЕНИЙ ФУНКЦИОНАЛЬНО-ДИФФЕРЕНЦИАЛЬНОГО ВКЛЮЧЕНИЯ С МНОГОЗНАЧНЫМИ ИМПУЛЬСНЫМИ ВОЗДЕЙСТВИЯМИ [1]

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Ключевые слова: функционально-дифференциальное включение; многозначные импульсные воздействия.

Аннотация: На множестве решений функционально-дифференциального включения с многозначными импульсными воздействиями рассмотрен вопрос о реализации расстояния в пространстве суммируемых функций от произвольной суммируемой функции до своих значений. Получены эффективные оценки решений задачи Коши.

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