SOME CONTROL PROBLEMS FOR LINEAR ABSTRACT FUNCTIONAL-DIFFERENTIAL SYSTEMS [1]

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The linear abstract functional differential equation [1] is the equation

$$\mathcal{L}x = f, \tag{1}$$

where $\mathcal{L} : \mathbf{D} \to \mathbf{B}$ is a linear bounded operator, $\mathbf{D}$ and $\mathbf{B}$ are Banach spaces such that $\mathbf{D}$ is isomorphic to the direct product $\mathbf{B} \times \mathbf{R^n}$. Let us denote by $\mathcal{J} = \{\Lambda, Y\} : \mathbf{B} \times \mathbf{R^n} \to \mathbf{D}$ an isomorphism and let $\mathcal{J}^{-1} = [\delta, r]$. Suppose that the so called principal boundary value problem $\mathcal{L}x = f, rx = \alpha$ is uniquely solvable for any $f \in \mathbf{B}$ and $\alpha \in \mathbf{R^n}$. In such a situation, the solution of the problem has the form $x = Gf + X\alpha$, where G is the Green operator, X the fundamental vector. Consider the control problem

$$\mathcal{L}x = Fu + f, \tag{2}$$

$$rx = \alpha,\ \ell x = \beta, \tag{3}$$

with a linear bounded operator $F : \mathbf{H} \to \mathbf{B}$, a linear bounded vector functional $\ell = [\ell_1, ..., \ell_m] : \mathbf{D} \to \mathbf{R^m}$ (it defines the aim of controlling), and the control u from a Hilbert space, $\mathbf{H}$. The equality $\lambda_i u = \ell_i GFu$ defines the linear bounded functional $\lambda_i : \mathbf{H} \to \mathbf{R}$. Let us denote by the same symbol λ_i the element of $\mathbf{H}$ that generates the functional λ_i, that is, $\lambda_i u = \langle \lambda_i, u \rangle$, where $\langle\ ,\ \rangle$ stands for the inner product in $\mathbf{H}$. Necessary and sufficient conditions for the solvability of the control problem (2),(3) are presented in the talk. Those are formulated in terms of the matrix $\Gamma = \{\langle \lambda_i, \lambda_j \rangle\}_{i,j=1,..,m}$.

Problem (2), (3) covers a wide class of control problems for ordinary differential systems, differential delay systems, some singular and impulsive systems. In the talk, some examples of the above problems are considered, and questions of developing contemporary computer-assisted techniques for the study of the control problems are discussed. The key idea of the computer-aided approach to the study of the problem (2), (3) with use of Computer Algebra Systems is based on the fact that the property of controllability of system (2) retains under small disturbance of the parameters. Thus effective tests of the controllability can be formulated in terms of the elements of a matrix Γ_0 that approximates Γ with a precision high enough. This requires a high precision of the approximation to X, G, F, ℓ within classes of so called «computable» functions and operators [1]. The questions of such approximations are discussed as well. Some applied control problems being reducible to the form (2), (3) are presented.

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REFERENCES

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Maksimov Vladimir Petrovich
Perm State University
Russia, Perm
e-mail: maksimov@econ.psu.ru

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ИНВАРИАНТЫ АФФИННОЙ ГРУППЫ ПРЯМОЙ В ПРОСТРАНСТВЕ МНОГОЧЛЕНОВ [1]

© **Н. А. Малашонок**

Рассматриваются орбиты и инварианты аффинной группы прямой $G = \{x \mapsto \alpha x + \beta\}$, действующей сопряжениями в пространстве многочленов $f(x)$.

Пусть V_n — пространство многочленов $f(x)$ степени $\leqslant n$ над полем $\mathbb{R}$:

$$f(x) = a_0 + a_1 x + \ldots + a_n x^n, \tag{1}$$

где $a_0, a_1, \ldots, a_n \in \mathbb{R}$ и переменная x пробегает $\mathbb{R}$. Пространство V_n имеет размерность $n+1$. Обозначим V_n^+, V_n^-, V_n^0 подмножества V_n, состоящие из многочленов $f(x)$ с $a_n > 0$, $a_n < 0$, $a_n = 0$, соответственно. Подмножество V_n^0 может быть естественным образом отождествлено с V_{n-1}. Таким образом, мы имеем фильтрацию

$$\mathbb{R} = V_0 \subset V_1 \subset V_2 \subset \ldots \subset V_n \subset \ldots . \tag{2}$$

Пусть G — группа аффинных преобразований φ прямой $\mathbb{R}$:

$$x \mapsto \varphi(x) = \alpha x + \beta, \tag{3}$$

где $\alpha, \beta \in \mathbb{R}$, $\alpha > 0$. Она действует в пространстве V_n многочленов $f(x)$ следующим образом:

$$T(\varphi)f = \varphi^{-1} \circ f \circ \varphi,$$

или

$$(T(\varphi)f)(x) = \frac{1}{\alpha} f(\alpha x + \beta) - \frac{\beta}{\alpha}.$$

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