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Dequantization of mathematical structures, tropical mathematics, and geometry [1]

© G. L. Litvinov, G. B. Shpiz
Moscow Center for Continuous Mathematical Education

A brief introduction to tropical and idempotent mathematics is presented. Geometrical applications are discussed

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Tropical mathematics can be treated as a result of a dequantization of the traditional mathematics as the Planck constant tends to zero taking imaginary values, see [1]–[4]. This kind of dequantization is known as the Maslov dequantization and it leads to a mathematics over tropical algebras like the max-plus algebra. The so-called idempotent dequantization is a generalization of the Maslov dequantization. The idempotent dequantization leads to idempotent mathematics over idempotent semirings. For example, the field of real or complex numbers can be treated as a quantum object whereas idempotent semirings can be examined as "classical" or "semiclassical" objects (a semiring is called idempotent if the semiring addition is idempotent, i. e. $x + x = x$).

Tropical mathematics is a part of idempotent mathematics. Tropical algebraic geometry can be treated as a result of the Maslov dequantization applied to the traditional algebraic geometry (O. Viro, G. Mikhalkin). There are interesting relations and applications to the traditional convex geometry [3].

In the spirit of N. Bohr's correspondence principle there is a (heuristic) correspondence between important, useful, and interesting constructions and results over fields and similar results over idempotent semirings. A systematic application of this correspondence principle leads to a variety of theoretical and applied results, see, e. g., [1]–[6].

In the framework of idempotent mathematics, a new version of functional analysis is developed from idempotent variants of basic theorems (e. g., of the Hahn-Banach type) to the theory of tensor products, nuclear operators and nuclear spaces in the spirit of A. Grothendieck as well as basic concepts and results of the theory of representation of groups in idempotent linear spaces, see, e. g., [1]–[4].

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Last time the Maslov dequantization and related dequantization procedures are applied to different concrete mathematical objects and structures, see, e. g., [3]–[6].

Examples:

1. The Legendre transform can be treated as a result of the Maslov dequantization of the Fourier-Laplace transform (V. P. Maslov).

2. If f is a polynomial, then a dequantization procedure leads to the Newton polytope of f. Using the so-called dequantization transform it is possible to generalize this result to a wide class of functions and convex sets, see [3].

3. An application of dequantization procedures to linear operators leads to spectral properties of these operators, see [6].

4. An application of a dequantization procedure to metrics leads to the Hausdorff-Besicovich dimension including the fractal dimension, see [6].

5. An application of a dequantization procedure to measures and differential forms leads to a notion of dimension at a point, see [6]. This dimension can be real-valued (e. g., negative).

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