## POSSIBILITIES FOR ALTERNATIVE USES OF THE «CALCULUS AND MATHEMATICA» APPROACH

Alkiviadis Akritas (University of Thessaly, Greece), Stelios Kapranidis (University of South Carolina Aiken), Athina Katalifou (Technical Educational Institution of Athens)

We believe that the «Calculus and Mathematica» (C&M) method of Davis Porta and Uhl is a powerful educational tool. In its standard format, however, it requires that the students are self-motivated and have practically unlimited access to computers. In this paper we present some of the results of our efforts to incorporate elements of the C&M approach when conditions are less than ideal with respect to computer availability and student motivation, discipline or computer literacy.

We believe that "Calculus and Mathematica" (C&M) of Davis Porta and Uhl is a well thought out and extremely valuable method that, when it is used properly, helps the students develop a most important intuitive understanding of basic calculus concepts. There are some difficulties, however, in implementing this method in the real classroom. For one thing, in its standard format, the method requires that the students have extensive access to computers furnished with Mathematica. Furthermore, it assumes that the students have the motivation, the time and the ability to follow the "guided search and discover" technique recommended by C&M. As a result, in cases where there are limited computer resources or the students are less motivated, have a limited time to devote to calculus, or have a weak math background the C&M method may not work very well.

Since we recognize the potential of the method but also some of the difficulties in its application, we decided to try, in a systematic way, to determine ways in which the students can benefit from the C&M approach even when the conditions for its application are less than ideal. In particular, we decided to explore (a) the possibility of using the C&M approach even with limited computer facilities, (b) the possibility to combine C&M with other calculus texts, and (c) the level and nature of assistance needed to help the students use C&M efficiently.

We think that our institutions provide an appropriate variety of conditions in which we can test the degree of success of different implementations of C&M. Specifically, the University of South Carolina Aiken has excellent computer facilities. Furthermore, many of our students have their own computers. Thus, computer access is not a problem. A very important characteristic of the student population is, however, that a substantial fraction of them are non-traditional students. These are older students, typically working people with families to support. They are, in general, highly motivated and intelligent students but they have a rather limited time for studying. Some also, when they start their studies, have some problems with basic math skills. In other words, the students in a class have quite heterogeneous backgrounds.

The University of Thessaly is a relative new university in Greece. The computer facilities are adequate. The classes are quite large. The students have very strong math backgrounds because, to be admitted to the university, they have to pass an entry exam that requires a rigorous preparation. Computer literacy is limited, however, and only about 5of the students own a computer, often an outdated one. In Greece, the calculus classes at the university level are traditionally taught in a more formal way than they are in the US. Thus, the C&M text needs to be supplemented by some formal definitions and proofs.

At the Technical Educational Institution of Athens the classes are large, comparable in size to the classes at the University of Thessaly. The computer facilities are somewhat limited. The students are expected to have weaker math backgrounds and lower computer literacy than university students because they are in general the students who scored lower that the students admitted to universities, such as the University of Thessaly, in the same entry exam. The curriculum is less theoretical or formal and more application oriented.

<sup>\*</sup>University of South Carolina Aiken

<sup>&</sup>lt;sup>†</sup>University of Thessaly, Greece

<sup>&</sup>lt;sup>‡</sup>"Technical Educational Institution of Athens"

Our results so far can be summarized as follows: Introductory lectures, carefully planned to be close to the C&M approach, are in general very helpful to the students. Quite commonly, students have problems with the Mathematica Code. We found that the best way to deal with this problem is to use the Mathematica code of the C&M text in our lectures. Essentially we found it necessary to teach Mathematica along with the conceptual development of the C&M approach. Also, making available to the students the complete answers to Mathematica based homework after their assignments have been graded improves their performance in the next assignment. Even when the students have very limited access to a computer lab, a presentation of the C&M electronic text to the students, by projecting it on a large screen, in a lecture format, seems to help them improve their intuitive understanding of calculus. Once the students develop an intuitive understanding of a new concept, a more formal definition or proof seems to make more sense to them

In our presentation we will provide specific examples of the types of lectures we use, of the handouts and other materials that we use in our classes, and the kind of support that our students need to avoid frustration and benefit from the C&M approach.

## CALCULUS AND THE RACE TRACK PRINCIPLE

Alkiviadis G. Akritas (University of Kansas EECS-IPS 415 Snow Hall, Lawrence, KS 66045, USA and University of Thessaly, Department of Theoretical and Applied Sciences, GR-38221 Volos, Greece)

Calculus and Mathematica (C&M) by Davis, Porta and Uhl ia a well thought-out method that, when used properly, gives students an intuitive understanding of, and a feeling for, all the major calculus concepts. It is comprised of the following four books: C&M / Derivatives, C&M / Integrals, C&M / Vector Calculus, and C&M / Approximation, known also as Books 1-4. In these books the authors advocate an explore-and-discover method for teaching the basic concepts of Calculus to undergraduate students. As indicated by the title of this series of books, Mathematica facilitates the exploration. One of the most astonishing thing encountered in Books 1, 2, and 4 is the use of the race track principle. This little known principle is elegantly used to prove or explain just about anything. Below we show examples of how this principle is used - to explain the round-off errors that appear in, say, handheld - to prove the second fundamental theorem of integration, and - to explain the series expansion of a function.