MSC 49J20,49J35, 47A75

Chattering control in infinite-dimensional spaces

© **M. I. Zelikin**
Lomonosov Moscow State University, Moscow, Russia

Lagrangian homogeneous manifolds in infinite-dimensional spaces for problems of optimal control of systems of partial differential equations with oscillating solutions are designed. The corresponding phase portrait has the structure of fiber bundle. The base consists of singular trajectories, fibers consist of trajectories with infinite number of switches on a finite time interval

Keywords: control systems, fiber bundles, partial differential equations, minimax problems, elliptic differential operators, Lagrangian homogeneous manifolds

We consider the control system

$$\frac{d^2z}{dt^2} + Bz = C(x)u,$$

where $t \in \mathbb{R}$, $x \in \Omega \subset \mathbb{R}^n$, $z : \mathbb{R} \times \Omega \to \mathbb{R}^k$. The function $C(x)$ takes values in $\mathbb{R}^k$. The control function u is a scalar, $|u| \leqslant 1$. The operator B is an elliptic differential operator in the domain Ω. As a boundary value problem we consider

$$z\big|_{t=0} = z_0(x); \quad \frac{dz}{dt}\Big|_{t=0} = z_1(x); \quad G = 0,$$

where G is a boundary operator such that B is self-conjugate; the spectrum of B is $\{\lambda_i\}$, eigenfunctions generate a basis in l_2. We seek the control function $u(x)$ that minimizes the functional

$$\int_0^\infty \int_\Omega |z(t,x)|^2 dx dt.$$

Expanding by eigenfunctions of B one obtains the problem of simultaneous stabilization of oscillators by the same scalar control that acts on each oscillator. So one has the control problem in the space l_2

$$\int_0^\infty \sum_{i=1}^\infty s_i^2(t)dt \to inf, \quad \frac{d^2s_i}{dt^2} + \lambda_i s_i = c_i u,$$

where $i = 1, 2, \ldots$, $|u| \leqslant 1$.

We proved:

(a) the necessary conditions for optimality (Pontryagin's maximum principle);

(b) the sufficiency theorem (the functional in question is convex);

(c) the existence and the uniqueness theorem.

We found the manifold S consisting of singular trajectories and proved a theorem on bundles:

Theorem 2 *Trajectories form a bundle over the base S with two-dimensional fibres filled by chattering trajectories, that is by trajectories reaching S in finite time interval with infinite number of switches.*

So we design chattering synthesis in infinite-dimensional space.

As an example we investigate the problem of mean square deviation of the elastic Timoshenko beam from the desired position. We consider the beam that is clamped at one end to a rotating disc and with free other end. We supposed that the beam is controlled by a bounded in absolute value torque acting on the disc. Using the theorem on bundles we found the optimal synthesis.