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Bravyi E.I. ON SOLVABILITY OF PERIODIC BOUNDARY VALUE PROBLEM AND DIRICHLET PROBLEM FOR SECOND ORDER FUNCTIONAL-DIFFERENTIAL EQUATIONS

The Dirichlet boundary value problem and the periodic boundary value problem for for some classes of linear second-order functional-differential equations are considered. Necessary and sufficient conditions of a unique solvability of the boundary value problem for all equations from these classes are obtained.

Key words: functional-differential equations; boundary value problems; periodic boundary value problem; solvability conditions; Dirichlet problem.

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ON CONNECTION BETWEEN CONTINUOUS AND DISCONTINUOUS HOMOGENIZED NEURAL FIELD EQUATIONS

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Key words: discontinuous Hammerstein equations; solvability; continuous dependence.

We study existence and continuous dependence of the solutions to the Hammerstein equation under the transition from continuous nonlinearities in the Hammerstein operator to the Heaviside nonlinearity in a vicinity of the solution, corresponding to the discontinuous nonlinearity case.

We consider the following generalization of the homogenized Amari neural field equation (see for example [1], [2])

$$\partial_t u(t,x,x_{\mathrm{f}}) = -u(t,x,x_{\mathrm{f}}) + \int\limits_{\Xi}\int\limits_{\mathcal{Y}} \omega(x-y, x_{\mathrm{f}}-y_{\mathrm{f}}) f_\beta(u(t,y))dy_{\mathrm{f}}dy, \qquad t>0,\ x\in\Xi\subseteq R^m,\ x_{\mathrm{f}}\in\mathcal{Y}\subset R^k, \tag{1}$$

parameterized by $\beta \in [0,\infty)$.

We assume that the functions involved in (1) satisfy the following assumptions:

(A1) For any $x_{\rm f} \in \mathcal{Y}$, the integration kernel $\omega(\cdot, x_{\rm f}) \in C^2(\Xi, R)$.

(A2) For any $x \in R$, the integration kernel $\omega(x, \cdot) \in L(\mathcal{Y}, \mu, R)$.

(A3) For $\beta = 0$, the Hammerstein nonlinearity is represented by the Heaviside function

$$f_0(u) = \begin{cases} 0, & u \leqslant \theta, \\ 1, & u > \theta \end{cases}$$

with some threshold value θ.

(A4) For $\beta > 0$, functions of the family $f_\beta : R \to [0,1]$ are non-decreasing, continuous, and satisfying the following convergence conditions with respect to the parameter β:
(i) $f_\beta \to f_{\widehat{\beta}}$ uniformly on R as $\beta \to \widehat{\beta}$, $\widehat{\beta} \in (0,\infty)$;
(ii) for any $\varepsilon > 0$, $f_\beta \to f_0$ uniformly on $R \setminus B_R(\theta, \varepsilon)$ as $\beta \to 0$.

If the stationary solution to (1) exists and does not depend on the fine-scale variable, it satisfies the following equation

$$\begin{aligned} u(x) &= \int\limits_{\Xi} \langle\omega\rangle(x-y) f_\beta(u(y))dy, \\ \langle\omega\rangle(x) &= \int\limits_{\mathcal{Y}} \omega(x, x_{\rm f})dx_{\rm f},\ x \in \Xi \subseteq R^m, x_{\rm f} \in \mathcal{Y}. \end{aligned} \tag{2}$$

We are interested here in one particular type of solutions, which possesses the following properties.

D e f i n i t i o n 1. Let $\theta > 0$ be fixed. We say that $u \in C^1(\Xi, R)$ satisfies the *θ-condition* if

(B1) there is a finite set of open bounded domains $\Theta_i \subset \Xi$ such that $u(x) > \theta$ on $\Theta = \bigcup\limits_{i=1}^{N} \Theta_i$;

(B2) for any point x of the boundary $\mathcal{B} = \bigcup\limits_{i=1}^{N} \mathcal{B}_i$ of Θ, it holds true that $u'(x) \neq 0$;

(B3) there exist $\sigma > 0$ and $r > 0$ such that $u(x) < \theta - \sigma$ for all $x \in \Xi \setminus B_{R^m}(\Theta, r)$.

The following theorem provides conditions for convergence of the stationary solutions u_β to (1), $\beta > 0$, (if these solutions exist) to the stationary solution u_0 to (1) at $\beta = 0$.

T h e o r e m 1. (Continuous dependence) *Let the assumptions* (A1) – (A4) *hold true, $\theta > 0$ be fixed and $u_0 \in C^1(R^m, R)$ satisfies the θ-condition. Then there exists $\varepsilon > 0$ such that for any (sufficiently large) closed $\Omega \subset R^m$, if we assume existence of solutions $u_\beta \in B_{C^1(\Omega,R)}(u_0, \varepsilon)$ to the equation (2) for any $\beta \in (0,1]$ $(\Xi = \Omega)$, then there exist a solution to (2) at $\beta = 0$ and it is a limit point of the set $\{u_\beta\}$. Moreover, if the solution of (2) at $\beta = 0$ $(\Xi = \Omega)$, say u_0, is unique then $\|u_\beta - u_0\|_{C^1(\Omega,R)} \to 0$.*

The next theorem provides a tool for proving existence of solutions to (2) for $\beta \in (0,\infty)$ using some knowledge about the solution to (2) at $\beta = 0$.

T h e o r e m 2. (Existence) *Let the conditions of Theorem 1 be satisfied, the set Ω and the constant ε be taken from Theorem 1. Assume that there exists solution $u_0 \in C^1(R^m, R)$ of (2) at $\beta = 0$, which satisfies θ-condition and which is unique in $\overline{B_{C^1(\Omega,R)}(u_0, \varepsilon_1)}$ $(\varepsilon_1 < \varepsilon)$, and $\deg(I - F_0, B_{C^1(\Omega,R)}(u_0, \varepsilon_1), 0) \neq 0$, where the operator $F_0 : B_{C^1(\Omega,R)}(u_0, \varepsilon_1) \to C^1(\Omega, R)$ is given by*

$$(F_0 u)(x) = \int\limits_{\Xi} \langle\omega\rangle(x-y) f_0(u(y))dy.$$

Then for any $\beta \in (0,1]$ *, there exists solution* $u_\beta \in B_{C^1(\Omega,R)}(u_0, \varepsilon_1)$ *to the equation* (2) *.*

These results can be applied for justification of the usage of the Heaviside Hammerstein nonlinearities in the frameworks of [1] and [2], where it appreciably simplified both theoretical and numerical investigations.

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Бурлаков Е., Поносов А., Виллер Й. О СВЯЗИ НЕПРЕРЫВНЫХ И РАЗРЫВНЫХ УСРЕДНЁННЫХ УРАВНЕНИЙ НЕЙРОПОЛЕЙ

Изучаются существование и непрерывная зависимость решений интегральных уравнений Гаммерштейна при переходе от непрерывной нелинейной части оператора Гаммерштейна к нелинейности типа Хевисайда в окрестности решения, соответствуюшего случаю разрывной нелинейной части.

Ключевые слова: разрывные операторы Гаммерштейна; разрешимость; непрерывная зависимость.

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